Sample Question Paper - 5
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
3. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
4. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study-based questions.

Section A

1. Solve the quadratic equation by factorization: **[2]**

   $ax^2 + (4a^2 - 3b)x - 12ab = 0$

   OR

   Find the roots of the following quadratic equation: $(x + 3)(x - 1) = 3\left(x - \frac{1}{3}\right)$.

2. A hemispherical bowl of internal radius 9 cm is full of liquid. The liquid is to be filled into cylindrical shaped bottles each of radius 1.5 cm and height 4 cm. How many bottles are needed to empty the bowl? **[2]**

3. The mean monthly salary of the 12 employees of a firm is Rs 1450. If one more person joins the firm 8 who gets Rs 1600 per month, what will be the mean monthly salary now? **[2]**

4. Find the 11^{th} term from the last term of the AP 10, 7, 4, ...,-62. **[2]**

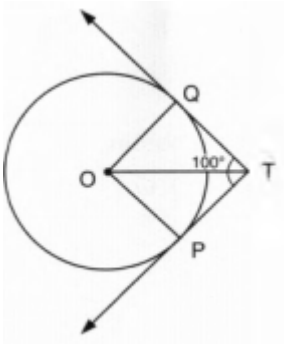
5. Compute the mode from the following series: **[2]**

Size	45 - 55	55 - 65	65 - 75	75 - 85	85 - 95	95 - 105	105 - 115
Frequency	7	12	17	30	32	6	10

6. A circle is touching the side BC of $\triangle ABC$ at P and touching AB and AC produced at Q and R respectively. Prove that AQ = $\frac{1}{2}$(perimeter of $\triangle ABC$). **[2]**

   OR

   Two tangents TP and TQ are drawn from an external point T to a circle with centre O as shown in Figure. If they are inclined to each other at an angle of 100°, then what is the value of $\angle POQ$?



Section B

7. If the sum of a certain number of terms starting from first term of an A.P. is 25, 22, 19,..., is 116. Find the last term. **[3]**

8. If at some time of the day the ratio of the height of a vertically standing pole to the length of its shadow on the ground is $\sqrt{3} : 1$, then find the angle of elevation of the sun at that time. **[3]**

OR

A moving boat observed from the top of a 150 m high cliff, moving away from the cliff. The angle of depression of the boat changes from 60^{o} to 45^{o} in 2 minutes. Find the speed of the boat.

9. Find the length of a tangent drawn to a circle with radius 5 cm, from a point 13 cm from the centre of the circle. **[3]**

10. Find the root of the quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$ by applying quadratic formula. **[3]**

Section C

11. Draw a circle of radius 4 cm. Take two points P and Q on one of its extended diameter each at a distance of 6 cm from its centre. Draw tangents to the circle from these two points P and Q. **[4]**

OR

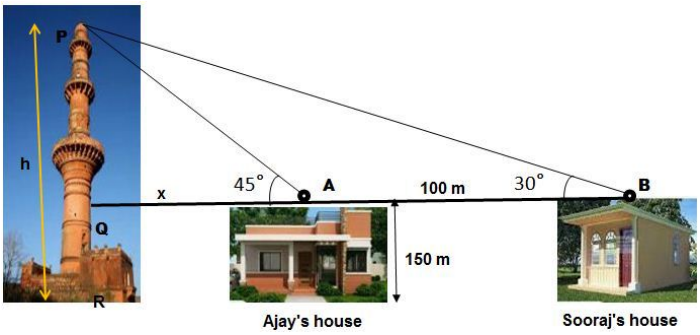
Draw a circle of radius 4 cm. From a point X, 9 cm away from the centre of the circle, draw two tangents to the circle. Also, measure the lengths of the tangents.

12. Find the mean marks of students for the following distribution: **[4]**

Marks	**Number of students**
0 and above	80
10 and above	77
20 and above	72
30 and above	65
40 and above	55
50 and above	43
60 and above	28
70 and above	16
80 and above	10
90 and above	8
100 and above	0

13. The houses of Ajay and Sooraj are at 100 m distance and the height of their houses is the same **[4]**

as approx 150m. One big tower was situated near their house. Once both friends decided to measure the height of the tower. They measure the angle of elevation of the top of the tower from the roof of their houses. The angle of elevation of ajay's house to the tower and sooraj's house to the tower are 45^o and 30^o respectively as shown in the figure.

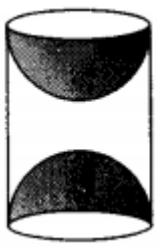


By using the above given information answer the following questions:

i. Find the height of the tower.

ii. What is the distance between the tower and the house of Sooraj?

14. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in the figure. **[4]**



If the height of the cylinder is 12 cm and its base is of radius 4.2 cm,

i. find the total surface area of the article.

ii. Also, find the volume of the wood left in the article.

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

Section A

1. Here we have, $ax^2 + (4a^2 - 3b)x - 12ab = 0$

   $\Rightarrow ax^2 + 4a^2x - 3bx - 12ab = 0$

   $\Rightarrow ax(x + 4a) - 3b(x + 4a) = 0$

   $\Rightarrow (ax - 3b)(x + 4a) = 0$

   $\Rightarrow x = \frac{3b}{a}$ or $-4a$ are two roots of the equation.

   OR

   $(x+3)(x-1) = 3\left(x - \frac{1}{3}\right)$

   or, $x^2 + 2x - 3 = 3x - 1$

   or, $x^2 - x - 2 = 0$

   or, $x^2 - 2x + x - 2 = 0$

   or, $x(x - 2) + 1(x - 2) = 0$

   or, $(x - 2)(x + 1) = 0$

   $\therefore x = 2, -1$

2. For hemispherical bowl

   Internal radius (r) = 9 cm

   $\therefore$ Volume$= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(9)^3 \text{cm}^3$

   For a cylindrical shaped small bottle

   Diameter = 3 cm

   $\therefore$ radius (R)$= \frac{3}{2}\text{cm}$

   Height (H) = 4 cm

   $\therefore$ Volume = $\pi R^2 H = \pi\left(\frac{3}{2}\right)^2(4)$

   $= 9\pi\text{cm}^3$

   Let n bottles needed, Then,

   volume e of n bottles = $9n\pi\text{cm}^3$

   According to the question,

   $9n\pi = \frac{2}{3}\pi(9)^3 \Rightarrow 9n = \frac{2}{3}(9)^3$

   $\Rightarrow n = \frac{2}{3}(9)^2 \Rightarrow n = 54$

   Hence, 54 bottles are needed to empty the bowl.

3. Sum of salary of 12 employees = n $\times$ mean

   $= 12 \times 1450$

   Salary of 13th person = Rs 1600

   Total salary of 13 employees $= 12 \times 1450 + 1600$

   Mean = $\frac{12\times1450+1600}{13}$ = 1461.538

   = Rs 1461.54

4. a = – 62, d = 3

   a_{11} = a + 10d

   = – 62 + 10(3)

   = – 32

5. Clearly, the modal class is 85 - 95, as it has the maximum frequency.

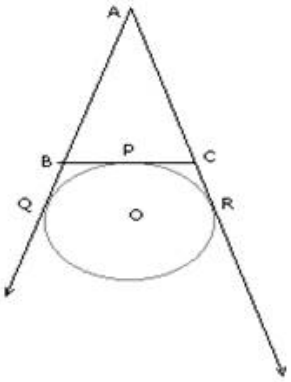
   $\therefore x_k = 85$, h = 10, $f_k = 32$, $f_{k-1} = 30$, $f_{k+1} = 6$

   Mode, $M_0 = x_k + \left\{h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$

   $= 85 + \left[10 \times \frac{(32-30)}{(64-30-6)}\right]$

   $= 85 + \frac{5}{7}$

= 85 + 0.71
= 85.71
Hence, mode = 85.71

6. 

We know that the two tangents drawn to a circle from an external point are equal.
∴ AQ = AR, BP = BQ, CP = CR
∴ Perimeter of △ABC = AB + BC + AC
= AB + BP + PC + AC
= AB + BQ + CR + AC [∵ BP = BQ, PC = CQ]
= AQ + AR = 2AQ = 2AR = [∵ AQ = AR]
= AQ = AR = $\frac{1}{2}$[Perimeter of △ABC]

OR

Consider the quadrilateral OPTQ. It is given that ∠PTQ = 100^o.
From the property of the tangent we know that the tangent will always be perpendicular to the radius at the point of contact. Therefore we have,

∠ OQT = 90^o

∠ OPT = 90^o

We know that the sum of all angles of a quadrilateral will always be equal to 360°.
Therefore,

∠PTQ + ∠OQT + ∠OPT + ∠POQ = 360^o

Let us substitute the values of all the known angles. We have,

$100^o + 90^o + 90^o$ + ∠POQ= 360^o

280 + ∠POQ= 360^o

∠POQ = 80^o

Therefore, the value of angle ∠POQ is 80°.

Section B

7. A.P. is given as: 25, 22, 19, ...
Sum of first n(say) terms = 116
Here a = 25 and d = 22 - 25 = 3
But $S_n = \frac{n}{2}[2a + (n-1)d]$
$\Rightarrow 116 = \frac{n}{2}[25 \times 2 + (n-1)(-3)]$
232 = n(50 - 3n + 3) $\Rightarrow$ 232 = (53 - 3n)n
$\Rightarrow$ 232 = 53n - $3n^2$ $\Rightarrow$ $3n^2$ - 53n + 232 = 0
$\Rightarrow$ $3n^2$ - 24n - 29n + 232 = 0

$$\begin{cases} \because 232 \times 3 = 696 \\ \therefore 696 = -24 \times (-29) \\ -53 = -24 - 29 \end{cases}$$

$\Rightarrow$ 3n(n - 8) - 29(n - 8) = 0
$\Rightarrow$ (n - 8)(3n - 29) = 0
Either n - 8 = 0, then n = 8
or 3n - 29 = 0 then 3n = 29 $\Rightarrow$ n = $\frac{29}{3}$
Which is not possible being a fraction

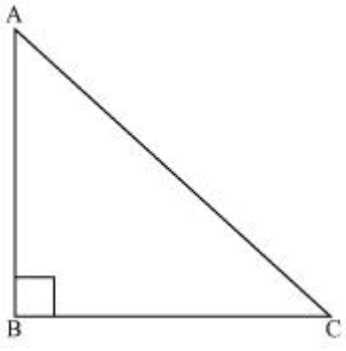
∴ Number of terms = 8

Now l = a_8 = a + (n - 1)d = 25 + (8 - 1) × (-3)

= 25 + 7(-3) = 25 - 21 = 4

∴ Last term = 4

8.



Let the pole be AB and length of the shadow be BC.

Given, the ratio of height of pole to the length of its shadow on the ground is $\sqrt{3} : 1$

So, $\frac{AB}{BC} = \frac{\sqrt{3}}{1}$

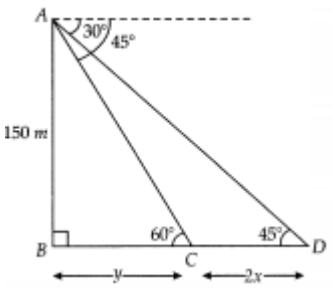
Consider the triangle ABC,

$\tan \angle ACB = \frac{AB}{BC} = \frac{\sqrt{3}}{1}$

$\angle ACB = 60^\circ$

Hence, the angle of elevation of the sun at that time is 60^o

OR



Let the speed of the boat be x m/min.

∴Distance covered in 2 minutes = 2x

∴ CD = 2x

Let BC = y

In △ABD,

$\frac{AB}{BC} = \tan 60^\circ$

$\Rightarrow \quad \frac{150}{y} = \sqrt{3}$

$\Rightarrow \quad y = \frac{150}{\sqrt{3}}$

$\Rightarrow \quad y = 50\sqrt{3}$.....(i)

In △ABD,

$\frac{AB}{BD} = \tan 45^\circ$

$\Rightarrow \quad \frac{150}{y+2x} = 1$

$\Rightarrow \quad y + 2x = 150$.....(ii)

Substituting the value of y from (i) in (ii) we get

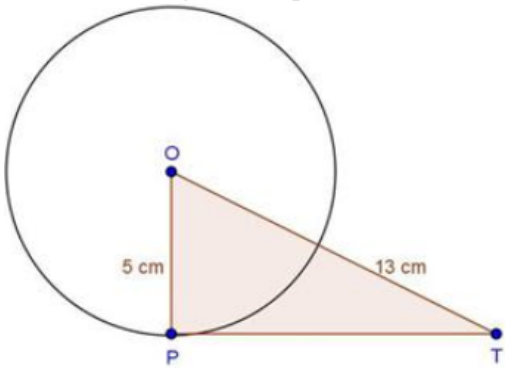
50√3 + 2x = 150

x = 75 - 25√3 = 25(3 - √3) m/sec

$= \frac{25(3-\sqrt{3})\times 60}{1000} = \frac{3}{2} \times (3 - \sqrt{3})$ km/min

9. According to question draw a circle with centre O and radius 5cm also given that T is any point outside of the circle.

Now,Since tangent at a point on the circle is perpendicular to the radius through the point.



Therefore, OP is perpendicular to PT.

In right triangle OPT, we have

$OT^2 = OP^2 + PT^2$

$\Rightarrow (13)^2 = (5)^2 + PT^2$

$\Rightarrow PT^2 = 13^2 - 5^2$

$\Rightarrow PT^2 = 169 - 25$

$\Rightarrow PT^2 = 144$

$\Rightarrow PT^2 = 12^2$

$\Rightarrow PT = 12$ cm

Hence, the length of a tangent is 12 cm.

10. $4x^2 + 4\sqrt{3}x + 3 = 0$

Comparing quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$ with the general form $ax^2 + bx + c = 0$, we get $a = 4$, $b = 4\sqrt{3}$ and $c = 3$

Putting these values in quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4a}}{2a}$

$x = \frac{-4\sqrt{3} \pm \sqrt{(4\sqrt{3})^2 - 4(4)(3)}}{2 \times 4} \Rightarrow x = \frac{-4\sqrt{3} \pm \sqrt{0}}{8}$

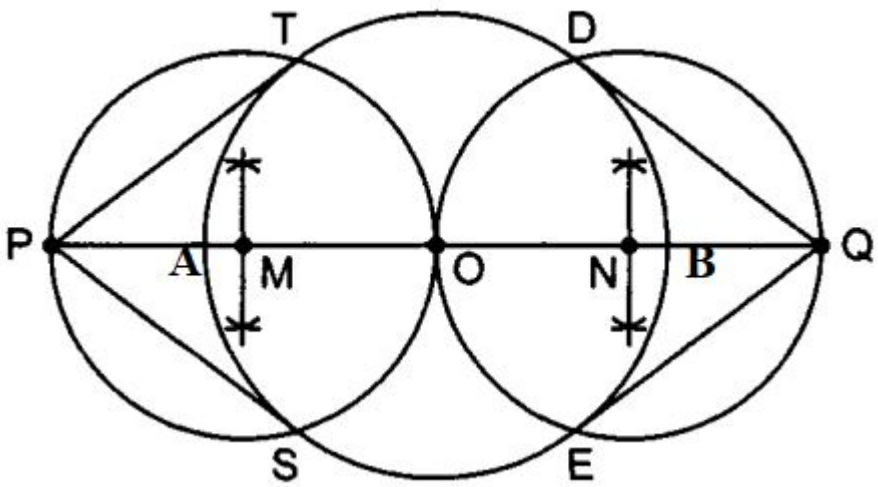
$\Rightarrow x = \frac{-\sqrt{3}}{2}$

A quadratic equation has two roots. Here, both the roots are equal.

Therefore, $x = \frac{-\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}$

Section C

11.



Steps of construction:

i. Draw a circle O as centre and of radius 4 cm.

ii. Draw AB as diameter of the circle.

iii. Take P and Q as two points on extended diameter AB such that OP = OQ = 6 cm.

iv. Draw perpendicular bisector of OP and OQ intersecting OP and OQ at M and N respectively.

v. With M as centre and OM as radius draw a circle intersecting the 1st circle at T and S.

vi. With N as centre and QN as radius intersecting the 2nd circle at D and E.

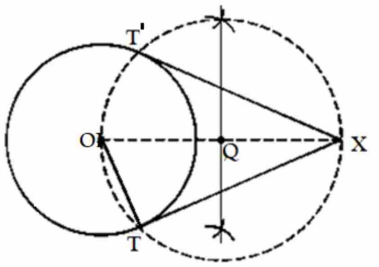
vii. Join PT, PS, QD and QE.

So, PT, PS, QD and QE are the required tangents.

OR

To construct a circle of radius 4 cm and a pair of tangents to the circle from a point, say X, 9cm from its centre.

To Find:The length of the tangents.



Construction:

Step 1. Draw a circle of radius 4 cm.

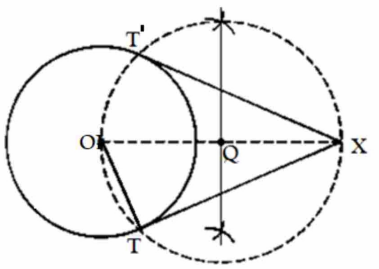
Step 2. Make a point X at a distance of 9 cm from O, and join OX.

Step 3. Draw a right bisector of X, intersecting OX at an angle of 90^0 at Q.

Step 4. Draw a circle to intersect the given circle at T and T`, taking Q as center and radius OQ = XQ.

Step 5. Join XT and X`T` to get the required tangents.

Hence, XT and X`T` are the tangents.



To find the length of the tangent, we know that OT⊥XT and △OXT is the right triangle.

Then,Since, OT = 4 cm and XO = 9 cm.

From △OXT, we have,

$XT^2 = OX^2 - OT^2 = 9^2 - 4^2$ [Using Pythagoras Theorem]

$= 81 - 16 = 65$

$XT = \sqrt{65}$ cm

∴ length of tangents = $\sqrt{65}$ cm

12. Calculation of mean:

Class	$\mathbf{f_i}$	$\mathbf{x_i}$	$u_i = \frac{x_i - a}{h}$	$\mathbf{f_i u_i}$
0 - 10	3	5	-5	-15
10- 20	5	15	-4	-20
20 - 30	7	25	-3	-21
30 - 40	10	35	-2	-20
40 - 50	12	45	-1	-12
50 - 60	15	55	0	0
60 - 70	12	65	1	12
70 - 80	6	75	2	12
80 - 90	2	85	3	6
90 -100	8	95	4	32

	$\Sigma f_i = 80$			$\Sigma f_i u_i = -26$

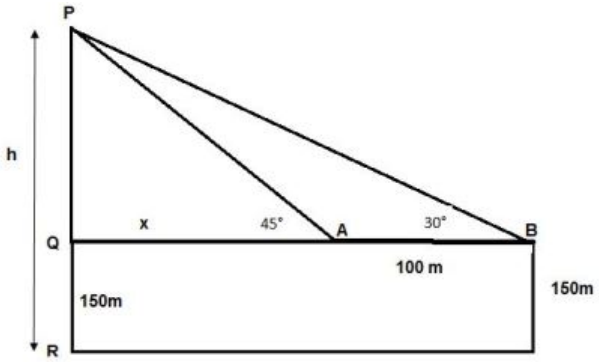
$a = 55\ and\ h = 10$

We know that, Mean = $a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h$

$=55 + \left(\frac{-26}{80}\right) \times 10$

$= 55 - 3.25 = 51.75$

13. The above figure can be redrawn as shown below:



i. Let PQ = y

In ΔPQA,

$\tan 45 = \frac{PQ}{QA} = \frac{y}{x}$

$1 = \frac{y}{x}$

x = y .....(i)

In ΔPQB,

$\tan 30 = \frac{PQ}{QB} = \frac{PQ}{x+100} = \frac{y}{x+100} = \frac{x}{x+100}$

$\frac{1}{\sqrt{3}} = \frac{x}{x+100}$

$x\sqrt{3} = x + 100$

$x = \frac{100}{\sqrt{3}-1} = 136.61\ m$

From the figure, Height of tower h = PQ + QR

= x + 150 = 136.61 + 150 = 286.61 m

ii. Distance of Sooraj's house from tower = QA + AB

= x + 100 = 136.61 + 100 = 236.61 m

14. Radius of the hemisphere, r = 4.2 cm. (It is same as radius of cylinder)

Radius of the cylinder, r = 4.2 cm

Height of the cylinder, h = 12 cm

Total surface area of the article

= curved surface area of cylinder + curved surface area of the 2 hemispheres

$= 2\pi rh + 2 \times 2\pi r^2 = 2\pi rh + 4\pi r^2$

$= 2\pi r(h + 2r) = \left[2 \times \frac{22}{7} \times 4.2 \times (12 + 2 \times 4.2)\right]\ cm^2$

$= (26.4 \times 20.4)cm^2 = 538.56cm^2$

Hence, Total surface area of article is 538.56 cm^2

Total volume of the wood left in the article

=volume of the cylinder - volume of the 2 hemispheres

$= \pi r^2 h - 2 \times \frac{2}{3}\pi r^3 = \pi r^2 \left(h - \frac{4}{3}r\right)$

$= \left[\frac{22}{7} \times 4.2 \times 4.2 \times \left(12 - \frac{4}{3} \times 4.2\right)\right]\ cm^3$

= (55.44 ×6.4)cm^3 = 354.816 cm^3.

Hence, total volume of article is 354.816 cm^3.